How Junior High Students Utilize Polya's Method to solve Open- Ended Algebraic Problem?

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ABSTRACT

This study aims to analyze the difficulties experienced by Junior High School (SMP) students in solving open-ended problems on algebra material and identify the contributing factors. Algebra is often considered an abstract and complex material because it involves symbols, variables, and mathematical operations that require deep conceptual understanding. The research gap emerges because there are still limited studies examining students' difficulties in the context of open-ended problems using Polya's problem-solving framework. This research employs a qualitative descriptive approach with a case study method. Research subjects consisted of several Junior High School (SMP) students in Boyolali who were purposively selected based on high, medium, and low academic ability levels. Data were obtained through observation, written tests, and in-depth interviews, then analyzed based on four problem-solving stages according to Polya, namely understanding the problem, devising a plan, carrying out the strategy, and looking back at the results. The research findings show that students' analytical thinking ability in solving open-ended algebra problems is still relatively low. The main obstacles found include difficulty understanding the problem's intent, limitations in formulating solution strategies, errors in algebraic manipulation, and lack of reflection on results. Contributing factors to these difficulties include the dominance of lecture methods, students' habit of working on single-answer problems, and weak mastery of basic algebraic concepts. This research recommends implementing problem-solving-based learning strategies and strengthening analytical thinking to improve students' flexibility and conceptual understanding in algebra learning.

Keywords: Open-Ended Problems; Algebra; Polya Framework; Analytical Thinking.

Introduction

Mathematics is a compulsory subject taught at every level of education, from elementary school to higher education (Hamzah, 2014). As a

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scientific discipline, mathematics has a systematic structure, starting from undefined elements, then defined elements, continuing to axioms or postulates, and finally
arriving at theorems (Muhni & Sumantri, 2015). The main function of mathematics learning is as a tool to understand and convey information through tables, graphs, or mathematical models. At the Junior High School (SMP) level, algebra material is often viewed as abstract because it involves symbols and variables. In addition, students are required to master arithmetic operations on algebraic forms based on the basic principles of integers.

Students' difficulties in understanding algebra generally stem from a lack of understanding of the concepts and principles underlying the material (Sundari & Wulantina, 2022). The concept of algebra becomes an important foundation for continuing learning at the next level. Other research also shows that during online learning, limitations in interaction and teacher guidance make the absorption of mathematics material less than optimal, so students experience difficulty following algebra learning, decreasing learning motivation and increasing student confusion in understanding basic algebra concepts (Ulfa Kurniyawati & Prastowo, 2021). If students do not understand the basic concepts and principles of algebra, they will experience difficulty in solving algebra problems, including problems with open-ended characteristics, which require deep understanding, reasoning ability, and answering more than one possible solution (Astrika et al., 2025).

This research differs from previous research because it systematically integrates the Dick and Carey learning development model in designing specific learning strategies to overcome students' difficulties with open-ended algebra problems. In addition, this research emphasizes problem-solving using Polya's approach which has not been widely adopted in the context of algebra learning in junior high schools. This approach not only assesses students' difficulties in general, but also examines how systematic models and strategies in problem- solving can be practically applied in the learning process to improve students' understanding and creativity in working on open-ended problems. Thus, this research provides an innovative contribution in combining problem-solving theory and learning development models to improve the effectiveness of algebra learning.

Based on the results of a pre-survey conducted by researchers through interviews with grade VIII mathematics teachers at one of the junior high schools, it was found that many students experienced difficulty in working on open-ended problems on algebra material. These difficulties include inability to define variables, constants, and coefficients, as well as in applying algebraic operations. This is exacerbated by students' weak understanding of the basic principles of integer operations which are the foundation of algebraic forms. Previous research also supports these findings, including Sugiarti and Retnawati (2019) who stated that students' difficulties in learning algebra are caused by weak mastery of basic concepts and principles. Similar findings were revealed by Hidayat et al. (2021) who emphasized that many students are not yet able to connect algebraic concepts with appropriate problem-solving strategies.

One approach that can be used to overcome these problems is George Polya's problem-solving theory which includes four stages, namely understanding the problem, planning a solution, carrying out the plan, and checking back on the results obtained. The urgency of mastering algebra and the ability to solve open- ended problems is very important in forming adaptive mathematical thinkers (Nila, 2008). Based on this background, researchers are interested in conducting research entitled "Analysis of Junior High School Students' Difficulties in Working on Open-Ended Algebra Problems." This research aims to identify the difficulties experienced by junior high school students in solving open-ended algebra problems, while revealing the contributing factors behind them. The method used is a qualitative descriptive approach with a focus on conceptual, procedural, and strategic difficulties (Sugiyono, 2020).

Algebraic forms are basic concepts in mathematics that are very important for junior high school students to understand because they involve variables, constants, and mathematical operations such as addition, subtraction, multiplication, and division. A variable is a symbol that represents an unknown or changeable value, while a constant is a fixed value that does not change in an expression. A coefficient is the numerical factor that multiplies a variable, for example in the term 2x, the number 2 is the coefficient. Understanding these elements is very important so that students can recognize and simplify algebraic forms appropriately (Kartika, 2018: 15).

In addition, the concept of like terms is also an important part of learning algebraic forms. Like terms are terms that have the same variables and powers, for example 2x and 5x, or $3y^2$ and $7y^2$. Only like terms can be added or subtracted, so grouping like terms facilitates the process of simplifying algebraic expressions. However, research shows that many junior high school students still experience difficulty in identifying and operating on like terms, which impacts their low algebraic thinking ability (Farida & Hakim, 2021: 1127).

Basic operations on algebraic forms include addition, subtraction, and application of the distributive property, such as in the expression $2(x+3) = 2x + 6$. Mastery of these operations is very important so that students can solve algebra problems correctly and efficiently. However, research findings reveal that junior high school students' mathematical understanding ability on algebraic form material is still relatively low, so they have difficulty applying algebraic operation concepts in solving everyday mathematical problems (Putra, 2018: 255). Thus, learning that focuses more on strengthening concepts and operational practice in algebra is very necessary.

In this context, the problem-solving theory proposed by George Polya

(1887-1985) becomes very relevant. George Polya (1887-1985) was a mathematician known as the "Father of problem solving". A problem can be understood as a situation that encourages someone to find a solution, although the solution steps are not directly available (Polya, 1985). If a student is able to solve problems given by the teacher well and correctly, then the problem is not a problem. Polya's problem-solving theory is very relevant in mathematics learning, especially in analyzing the difficulties of Junior High School (SMP) students in working on open-ended algebra problems. This approach helps students to understand problems, plan solutions, solve, and check back on results systematically (Saedi et al., 2020).

The first step is to understand the problem, namely identifying what is known, what is asked, as well as variables and relationships between variables in an algebraic context. After that, students develop a solution plan by choosing the right strategy and appropriate algebraic formula. The third step is to carry out the plan by applying algebraic operations carefully and logically. Finally, students check back on the solution obtained, ensure the answer makes sense and try other methods for verification, while reflecting on the process and looking for more efficient ways in the future.

Analysis of students' problem-solving ability based on Polya's theory on open-ended problems is done by looking at four main indicators: understanding the problem, planning the solution, carrying out the plan, and looking back. This method describes in detail the students' thinking process in solving problems that do not have a single answer. With this approach, the depth of students' understanding and creativity in mathematics can be assessed effectively. The stage of understanding the problem requires students to recognize what is known and sought, as well as the variables involved. Next, planning involves developing strategies and selecting appropriate formulas. Implementation requires applying steps carefully and logically, while looking back ensures the solution is correct and makes sense. In addition, this stage provides students the opportunity to reflect on the process and improve their problem-solving ability in the future (Agustina et al., 2021).

Method

This research uses a descriptive qualitative approach that aims to describe in detail the phenomenon of students' difficulties in working on algebraic form problems in the field. According to Moleong (2017), descriptive qualitative research aims to understand phenomena deeply and holistically in a natural context. Data is collected and analyzed carefully to reveal aspects, considering internal and external factors causing students' difficulties. This approach was chosen because it allows researchers to explore students' learning experiences deeply in a natural context, so that the main focus of research is understanding students' thinking processes and responses to various types of mathematical problems (Miles et al., 2014). The use of case study methods allows researchers to trace deeply students' difficulties in learning mathematics with the aim of obtaining a clearer and more complete picture of the problems they face.

Data was obtained through observation of learning activities, written test results, and in-depth interviews with students as research subjects. Research subjects consisted of several Junior High School (SMP) students in Boyolali who were selected based on variations in academic ability levels using purposive sampling technique, namely students with high, medium, and low abilities. Purposive sampling technique is a sample determination technique with certain considerations made by researchers to select subjects who are considered to best understand the problem being studied (Sugiyono, 2017). Researchers hope to obtain a more complete picture of the forms and varieties of students' difficulties in understanding algebraic forms by using subjects from these three ability levels.

This method is in accordance with Sugiyono's (2020) explanation which states that descriptive qualitative research aims to study phenomena in natural conditions with the researcher as the main instrument and using triangulation techniques to increase data validity. Data analysis was carried out through stages: data reduction, data presentation, and drawing conclusions. Polya's problem- solving theory divides the mathematical problem-solving process into four main stages: understanding the problem, planning the solution, carrying out the plan, and looking back at the results (Polya, 1985). Based on these four stages, indicators of students' difficulties in working on algebraic form problems are shown in Table 1:

Table 1: Indicators of Students' Difficulties in Working on Algebraic Form Problems

Problem-Solving Stage	Difficulty Indicators
1. Understanding The Problem	a. Difficulty understanding the content of the problem or problem statement b. Misinterpreting algebraic terms or symbols c. Unclear about what is actually being asked in the problem
2. Devising a Plan	a. Difficulty choosing the right strategy or method to solve the problem b. Unable to connect the problem with concepts that have been learned c. Lack of experience in dealing with similar problems
3. Carrying out the Plan	a. Making errors in algebraic manipulation (e.g., wrong arithmetic operation, wrong in managing algebraic forms) b. Calculation errors c. Difficulty following logical steps in solution

4. Looking Back	a. Not rechecking the obtained answer b. Unable to verify the correctness of the solution c. Lack of reflection on the problem-solving process that has been done

Results

The results of this research show that the problem-solving ability of junior high school students in solving open-ended problems on algebra material is still relatively low. This can be seen from the average score obtained by students who have not reached the minimum completeness criteria. Several students were seen still experiencing difficulty in understanding the intent of the problem, especially in identifying relevant information. During the interview process, students admitted to being less accustomed to working on problems that have more than one correct answer. They were more often trained to work on closed problems so they were not accustomed to open-ended type problems. This shows weakness in problem-solving ability in the aspect of providing logical reasons. Thus, it can be concluded that open-ended type problems require more complex thinking skills.

Problem 1

A rectangle's area is known to be 24 cm^2 and its perimeter is 20 cm. Determine the length and width of that rectangle!

Problem 2

A rectangle's area is known to be 40 m^2 and its perimeter is 26 m. Determine the length and width of that rectangle!

Problem 3

If 2 (x + 3) = x + 10, find the value of x!

Figure 1. Linear Algebra Problems

1. Understanding the Problem

In the aspect of understanding the problem, all three students (A, B, and C) in general were able to identify basic information given in the first problem, namely the area and perimeter of a rectangle. However, only students A and B understood that the purpose of the problem was to find the length and width values consistently.

a. Student A: Showed appropriate understanding. He was able to identify known information (area and perimeter) and information sought (length and width).
b. Student B: Also understood the purpose of the problem, although not fully paying attention to both important pieces of information, namely area and perimeter, because on one occasion only used area.
c. Student C: Did not show adequate understanding. He appeared

confused and tended to guess without strategy. This shows inability to identify important elements in the problem, especially in problems 2 and 3.

In problem 3, only student A consistently understood that the purpose of the problem was to solve the equation and find the value of x. Students B and C showed errors in understanding algebraic concepts.

2. Planning the Solution

The planning strategy is seen from how students seek solutions from known information.

a. Student A: Showed good planning. He tried several pairs of length and width whose multiplication results matched the area, then checked whether the pair also matched the perimeter. This shows logical and systematic thinking.
b. Student B: Remembered the area formula but forgot to consider perimeter, showing a less comprehensive plan. In problem 2, he only used one requirement (area), not both (area and perimeter).
c. Student C: Did not have a clear solution plan. He used a trial-and-error approach and appeared not to know the formula or concept to be used. In problem 3, he even misinterpreted algebraic form, indicating conceptual error.

3. Carrying out the Strategy

At the implementation stage, differences in students' mathematical manipulation abilities are clearly visible:

a. Student A: Executed the procedure correctly, especially in problem 3. He succeeded in solving the linear equation with correct steps:
b. $2(x + 3) = x + 10 \rightarrow 2x + 6 = x + 10 \rightarrow x = 4$
c. In problems 1 and 2, he could also multiply and calculate perimeter to ensure his answer was correct (although not yet evaluating alternative possibilities thoroughly).
d. Student B: Made an error in problem 3 because he was wrong in simplifying the equation. He divided directly without completing the algebraic form, which produced the wrong x value. In problems 1 and 2, he could answer the area correctly but ignored perimeter verification.
e. Student C: Did not succeed in carrying out the strategy correctly. In problems 1 and 2, he guessed numbers without calculation. In problem 3, he combined letters (x and y) like numbers, which is a fundamental algebraic concept error.

4. Evaluating Results

The evaluation or verification stage is an important step to assess whether students can evaluate back the process and results obtained:

a. Student A: Showed awareness to recheck the answer in problem 3 by substituting back the x value into the initial equation, which produced a true statement (14 = 14). This shows good evaluation ability. However, in problems 1 and 2, he did not evaluate whether there were other possible pairs of length and width.

b. Student B: Did not evaluate his work results. He did not check whether the obtained result matched the perimeter requirement. In problem 3, he did not realize that the result x = 3 did not satisfy the equation.
c. Student C: Did not perform verification at all. This may be because from the beginning there was no strategy or result that could be checked logically. This indicates weakness in reflection on mathematical thinking processes.

Discussion

Research results show that junior high school students' ability to solve open-ended problems on algebra material is still relatively low. Most students have not been able to reach minimum completeness criteria and experience difficulty in understanding the intent of the problem and identifying important information. This condition illustrates that students are not yet accustomed to facing problems that have more than one correct solution. These findings are in line with the opinion of Wardhani and Rumiati (2011) who stated that open-ended problems require higher-order thinking skills, such as the ability to develop various strategies and solution methods. Students' difficulties in this context show low creative thinking ability and flexibility in understanding complex mathematical problems.

Based on the analysis results of each problem-solving ability indicator according to Polya (1985), students' lowest achievement is in the indicator of providing explanations and logical arguments. Most students only write the final result without showing the underlying thinking process. In interviews, students admitted to being confused when asked to explain reasons or solution steps, because they were accustomed to closed problems that focus on a single answer. This phenomenon is in line with Sumarmo's (2013) findings which emphasize that mathematical problem-solving is not only about obtaining the final result, but also the ability to explain the reasons for each step. Thus, strengthening logical and argumentative reasoning aspects in mathematics learning becomes an urgent need so that students can think rationally and systematically.

Interestingly, some students who have high interest in mathematics are able to solve open-ended problems with varied strategies and logical explanations. These findings support research by Rachmantika & Wardono (2019) which stated that problem-solving and critical thinking abilities can be improved through active learning that provides space for idea exploration and self-reflection. In line with Hudojo's (2005) opinion, meaningful mathematics learning occurs when students actively construct their knowledge through thinking activities and finding patterns. In addition, Bruner (1961) emphasized that learning that allows students to discover concepts themselves will strengthen understanding and knowledge retention. Thus, activity-based learning (active learning) is important to apply because it provides opportunities for students to build knowledge through discussion, exploration, and independent idea communication.

In addition to internal factors such as interest and motivation, students' difficulties in problem-solving are also influenced by external factors, especially teaching methods that are still dominated by one-way lectures. These results are in line with research by Meilani et al. (2023) which shows that monotonous learning makes students less trained to face problems that require in-depth analysis. Polya (1973) emphasized that problem-solving success not only depends on concept mastery, but also on reflective thinking habits through stages of understanding the problem, planning strategies, implementing, and checking back on results. In line with this, Schoenfeld (1985) explained that mathematics learning must emphasize thinking strategies and reasoning, not just mechanistic solutions. Therefore, learning innovation based on problem-solving becomes important to foster analytical ability and thinking flexibility of junior high school students.

In the context of 21st-century education, critical thinking and problem- solving abilities are main competencies that must be developed in schools. BSNP (2010) emphasizes the importance of mastering 4C skills (critical thinking, communication, collaboration, creativity) as part of modern student competencies. Fauzan (2022) also found that learning based on small group discussions can increase students' confidence and higher-order thinking abilities. This opinion is reinforced by Marzano (2017) who stated that students who are active in collaborative learning processes are more able to develop conceptual understanding and deep self-reflection. Thus, teachers need to be trained to design and implement open-ended problems that match students' ability levels and encourage collaborative and explorative learning atmospheres, so that students become accustomed to critical thinking, logical argumentation, and effective mathematical communication.

Overall, the results of this research emphasize that junior high school students' problem-solving ability on algebra material still needs to be improved through learning oriented toward higher-order thinking development. These results are in line with Astrika et al. (2025) who found that open-ended problems have the potential to train analytical ability, but their effectiveness depends heavily on learning strategies implemented by teachers. Therefore, collaboration is needed between teachers, students, and curriculum in consistently integrating problem-based learning so that students are not only proficient in calculating, but also able to think logically, reflectively, and creatively in facing mathematical problems.

For further research development, it is recommended that researchers emphasize more on the development and application of learning strategies that ensure students' active involvement in mathematical problem-solving continuously. Research can also expand the scope by involving variations of different classes and schools to obtain more representative data and stronger result generalization. In addition, in-depth evaluation needs to be conducted on the role of teachers in managing higher-order thinking learning and its influence on students' learning outcomes holistically. Thus, subsequent research results can provide practical and applicable recommendations that can be used by educators and policymakers in improving the quality of mathematics learning at the junior high school level.

Conclusion

The conclusion from the results of this research is that junior high school students' ability to solve open-ended problems on algebra material is still relatively low, especially in terms of understanding problems, developing solution strategies, and providing logical reasons for their answers. This difficulty is caused by students' habits of more often working on closed problems and teaching methods that dominantly use lectures and routine problem exercises without requiring in-depth analysis. Only a small portion of students who are accustomed to explorative problems and group discussions are able to show better problem- solving abilities.

Therefore, learning strategies are needed that emphasize conceptual understanding, logical reasoning, and consistent problem-solving habituation, as well as a classroom environment that supports discussion and idea exploration. Teachers also need to be trained in designing appropriate open-ended problems and creating a safe learning atmosphere so that students' problem-solving skills can develop optimally. This research emphasizes the importance of integrating higher-order thinking skills in daily mathematics learning so that students are not only proficient in calculating but also able to think logically and reflectively in facing mathematical problems.

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